Time-Resolved Phase-Space Distributions for Light Backscattered from a Disordered Medium

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We demonstrate time-resolved measurement of optical phase-space distributions as a new probe for investigating the propagation of light in disordered media. Phase-space techniques measure the joint transverse position and momentum distribution of the scattered light, and are sensitive to the spatially varying phase and amplitude of the field. Using this method we investigate light backscattered from a random medium. The measurements indicate that the weakly localized component is a phase conjugate of the incident light field. A new model of backscatter, based on Wigner phase-space distributions, elucidates the spatial and angular behavior of the localized and unlocalized components.

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Multiple scattering of light in disordered media continues to be of great interest in a wide variety of disciplines. For example, in biomedical imaging, multiple scattering plays an essential role in limiting the spatial resolution obtained in tomographic applications [1]. Of particular interest is the detection of fundamental features of the scattered light which depend on the phase and amplitude of the field, such as the potential observation of Anderson localization [2]. This refers to a strong reduction of the diffusion coefficient in a disordered medium due to the interference of all of the scattered waves. A precursor to Anderson localization is weak localization [3] which is manifested for optical fields [4–7] by an increase in the intensity of the retroreflected light as compared to that in other directions. This enhancement arises from the interference of forward and time-reversed field paths, and is not described by radiative transport theory, which neglects the interferences. By contrast, the interferences can be rigorously described by a Wigner phase-space distribution, i.e., the *joint* position-momentum distribution [8], which is sensitive to the spatially varying phase and amplitude of the scattered field. Hence, it is important to develop methods which measure the dynamical evolution of Wigner phase-space distributions for the scattered field.

In this Letter, we demonstrate measurement of time-resolved phase-space distributions for weakly localized light backscattered from a random medium. We develop a new model of backscatter, based on Wigner distributions, which clearly shows that the weakly localized component of the backscattered light does not exhibit diffusion of the spatial variable and narrows in momentum as the time increases, in contrast to the unlocalized contribution which diffuses in space and has a broad momentum distribution. Remarkably, we detect no wave front curvature in the weakly localized light, although curvature is measured for the incident light. We show that this result is a consequence of time reversal and phase conjugation of the incident field.

The optical phase-space distributions measured in our experiments are directly related to the Wigner distribution, which in turn is Fourier transform related to the *mutual coherence* function of the optical field, $\langle \mathcal{E}^*(x)\mathcal{E}(x')\rangle$ [8]. The mutual coherence function is sensitive to the spatially varying phase and amplitude of the scattered optical field and to the interferences which cause localization. Direct measurement of the mutual coherence function has been achieved by shearing interferometry [9] and has been used to explore decoherence in a multiple scattering medium [10]. Soon after optical phase-space distributions were first obtained from intensity measurements in multiple planes [11], they were also obtained using heterodyne detection [12]. The heterodyne technique was first applied to examine multiple diffractive scattering of coherent light transmitted through a disordered medium [13]. By using an inexpensive broadband light source [14,15], the heterodyne method permits the selection of optical path lengths in the medium, effectively enabling *time-resolved* phase-space measurement. This method has been used to study the dynamical evolution of light transmitted through a random medium, isolating phase-space distributions for low-order scattering [16]. Unfortunately, a theoretical treatment of the measured time-resolved phase-space distributions does not yet exist for transmitted light. On the other hand, backscattering of light from disordered media and the phenomenon of weak localization, or enhanced backscatter, is an extensively studied topic both in theory [6,17,18] and experiment [4,5,7,19–21]. These previous works investigated only the position-integrated angular distribution of the backscattered light. However, we find that the previous theory can be readily extended to yield the time-resolved phase-space distributions of the backscattered light for comparison to our measurements.

We begin by describing a simple model of the Wigner distribution for light backscattered from a disordered medium, where the input field has arbitrary mutual coherence. For a wave field $\mathcal{E}$ varying in one spatial

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dimension, the Wigner phase-space distribution is given by [8,22]

$$W(x,p) = \int \frac{dq}{2\pi}\, e^{iqx} \left\langle \mathcal{E}^*\left(p - \frac{q}{2}\right)\mathcal{E}\left(p + \frac{q}{2}\right)\right\rangle, \tag{1}$$

where x is a transverse position, p is a transverse wave vector (momentum) in the x direction, and $\langle\cdots\rangle$ denotes a statistical average for the random medium.

The Wigner distribution for the backscattered light is determined by extending a previous calculation of the angular distribution by Okamoto and Asakura [17]. In the geometry of the backscattering problem, the light field component propagating with transverse momentum p_i first scatters at a point r_i, eventually exiting the medium with transverse momentum p_f at the final scattering point r_f. The total field component $\mathcal{E}(p_f)$, for final momentum p_f, is found by summing over all possible combinations of initial and final scattering positions, all incident transverse momenta p_i and all possible scattering paths within the medium. Reference [17] determines the angular distribution of the backscattered light, which for small angles is essentially the transverse momentum distribution $\langle\mathcal{E}^*(p_f)\mathcal{E}(p_f)\rangle$. To determine the Wigner function, we calculate instead the transverse momentum *coherence* $\langle\mathcal{E}^*(p_f - \frac{q}{2})\mathcal{E}(p_f + \frac{q}{2})\rangle$. In addition, we assume a wide band light source for the heterodyne scheme, permitting selection of the propagation time t in the medium.

Following Okamoto and Asakura [17], we assume that the total phase for different paths in the medium is random, so that the only contributions to the momentum space coherence arise from single paths and time-reversed pairs. For a homogeneous random medium, the sum over paths can be described by a probability distribution that depends only on the distance between the initial and final scattering points. As in Ref. [17], we assume that, for a sufficiently dense random medium and a long enough propagation time, the probability for propagation from $\mathbf{r_i}$ to $\mathbf{r_f}$ in a time t can be found using a diffusion approximation [18]. By using these results, the transverse Wigner distribution for the backscattered light at the face of the sample takes a very simple form:

$$W_B(x_f, p_f, t) \propto \frac{1}{4\pi D t}\int dx_i \int dp_i\, W_0(x_i,p_i)\exp\left(-\frac{(x_f - x_i)^2}{4Dt}\right) + \int dp_i\, W_0(x_f,p_i)\exp[-Dt(p_f + p_i)^2], \tag{2}$$

where position and momentum integrations are over the transverse directions x and y, with the y dependence (and integrals) suppressed for simplicity. Here $W_0(x_i,p_i)$ is the Wigner distribution of the input field, D is the diffusion constant given by $D = v l^*/3$, where v is the speed of light in the medium, and l^* is the transport mean free path [18].

Equation (2) illustrates the essential features of weak localization. The second term is readily identified as the weakly localized component since the spatial variable does not exhibit diffusion, i.e., $x_f = x_i$. This term arises from the interference of forward and time-reversed paths in the same coherence area and peaks in the backward direction where $p_f \simeq -p_i$. The localized component narrows in momentum due to the diffusive increase in the coherence area Dt as the time t spent in the medium increases. In contrast, the first term represents the unlocalized contribution which diffuses in space, i.e., $x_f \neq x_i$, and has a broad momentum distribution. Note that $\int dp_i W_0(x_i,p_i) = \langle|\mathcal{E}_0(x_i)|^2\rangle$ is the position distribution of the incident field $\mathcal{E}_0$, so that the broad component is independent of momentum in the small angle approximation.

In the experiments, we characterize the Wigner phase-space distribution of the backscattered optical field by using a simple heterodyne detection scheme. We measure the *mean square* beat amplitude S_B which is proportional to the squared magnitude of the spatial overlap integral of the local oscillator (LO) and signal fields in the detector planes (Fig. 1). Using Fourier optics, it is straightforward to show that [12]

$$S_B(d_x,d_p) \propto \int dx\, dp\, W_{\mathrm{LO}}(x - d_x, p + kd_p/f) \times W_B(x,p)\,. \tag{3}$$

Here $W_B(x,p)$ $[W_{\mathrm{LO}}(x,p)]$ is the Wigner distribution given by Eq. (1) of the backscattered (LO) field in the input plane of lens L2 (L1) of focal length $f = 6$ cm. The y integration is suppressed for simplicity. Equation (3) shows that the signal Wigner distribution is smoothed by that of the LO [23]. For a Gaussian LO beam, the position resolution is determined by the diameter of the LO beam while the momentum resolution is determined by the corresponding diffraction angle. The relative position d_x between the LO and signal fields is scanned by translating

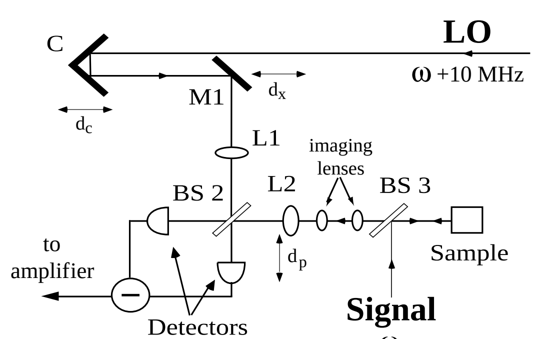

FIG. 1. Scheme for heterodyne measurement of time-resolved phase-space distributions for backscattered light. A broadband source is used to obtain timing resolution.

mirror M1 in the LO input path. The relative momentum $-kd_p/f$, where $k = 2\pi/\lambda$, is scanned by translating the signal beam input lens L2 by a distance d_p [12].

A conventional $4f$ imaging system [24] is used to reproduce the phase and amplitude of the backscattered field, and hence the backscattered Wigner distribution, in the input plane of lens L2. The LO and signal fields are combined at beam splitter BS2, and the rms beat amplitude is measured with an analog spectrum analyzer tuned to the beat frequency, 10 MHz. The output of the spectrum analyzer is squared in real time with a low noise multiplier, the output of which is sent to a lock-in amplifier. The lock-in output is directly proportional to $S_B(d_x, d_p)$, as given in Eq. (3).

For broadband light sources, the optical paths between the LO and signal arms must be matched to within the longitudinal coherence length of the source in order to measure a beat signal. This is achieved by including a corner cube, C, in the LO path which is scanned by a distance d_c. Scanning mirror M1 and lens L2 (Fig. 1) also introduces changes in the optical path lengths which must be compensated to keep the relative path difference constant. The difference between the LO and signal paths, when the sample is replaced by a mirror, is $\Delta = 2d_c + d_x - d_p^2/(2f) + d_x d_p/f$ [14]. In the current experiments, a superluminescent diode from the Anritsu Corporation, operating at 85 mA with an output power of 1.5 mW, is used to provide both the signal and LO beams. The center wavelength is $\lambda = 852$ nm and the bandwidth is 10.3 nm (full width at half maximum). The beat intensity drops by $1/e$ at $\Delta = 26.4$ μm, which determines the path length and temporal resolution [14].

Solutions of 0.5 μm diameter polystyrene spheres ($n = 1.59$) are used as samples to obtain broad angular scattering distributions. A glycerol/water mixture ($n_0 = 1.36$) is used to provide neutral buoyancy for the spheres. For a scatterer concentration $\rho = 1.1 \times 10^{12}$ cm^3, we obtain a scattering mean free path of $l = 13.8$ μm. Using the Mie solution, the transport mean free path is estimated to be $l^* = 53.1$ μm. In our experiments, the path length in the medium is large compared to the scattering mean free path l, so that the diffusion approximation is valid for all except the shortest time delays.

Time-resolved optical phase-space distributions were measured for light backscattered from the turbid sample. The effective time delay $t = \Delta/c$ is determined by the path delay Δ between the LO and signal beams in air, which ranges from 0–1 mm. Figures 2(a)–2(c) show the measured distributions for $vt = 7$ μm, 0.15 mm, and 0.44 mm, respectively, where v is the speed of light in the medium. For $vt = 7$ μm, the time delay is less than the temporal resolution for our source. A distinct variation is observed in Figs. 2(a)–2(c). For the longer time delays, the momentum distribution of the central localized component narrows, and a bright spot appears at zero transverse momentum and zero position. This is the weakly localized component. The gray bands extending in momentum correspond to the broad unlocalized component.

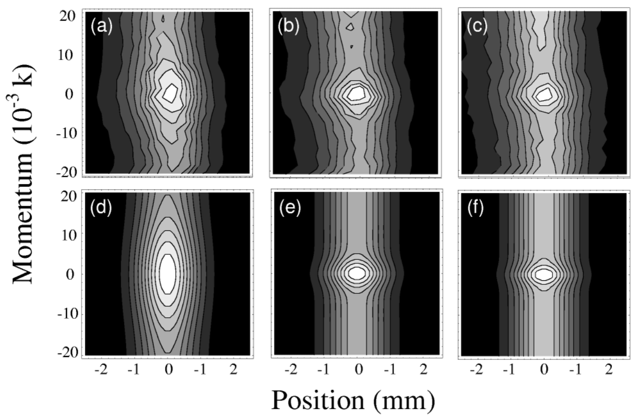

FIG. 2. Phase-space distributions for light backscattered from a random medium for different time delays. The broad gray bands extending in momentum are the diffuse component, while the bright central feature is the weakly localized component which narrows in momentum as the time delay is increased. The time delay $t = \Delta/c$ is determined by the path delay Δ between the LO and signal beams in air. (a)–(c) These panels show the measured distributions for $vt = 7$ μm, 0.15 mm, and 0.44 mm, respectively, where v is the speed of light in the medium. The (d)–(f) panels show the corresponding predictions using our model. Note that the momentum is given in units of the optical wave vector, $k = 2\pi/\lambda$ in air.

We model the optical phase-space distributions by calculating the mean square beat amplitude S_B [Eq. (3)] using the approximate Wigner function of the backscattered light,W_B [Eq. (2)]. The statistical average over the random medium is already included in W_B. However, for a broadband source, the beat signal arises from the correlation *between the LO and input* fields. The corresponding correlation between W_{LO} and W_B is evaluated by writing W_{LO} in terms of $\mathcal{E}_{\mathrm{LO}}(x)$ and writing W_0 in Eq. (2) in terms of the input field $\mathcal{E}_0(x)$. The correlation is determined from the cross spectral density which, in the notation of Ref. [25], is given by

$$\langle \mathcal{E}^*_{\mathrm{LO}}(x)\mathcal{E}_0(x')\rangle \propto \exp\left[-\frac{x^2 + x'^2}{4\sigma_s^2}\right]\exp\left[-\frac{(x - x')^2}{2\sigma_g^2}\right] \times \exp\left[\frac{ik}{2R}(x^2 - x'^2)\right]. \quad (4)$$

In this expression, we introduce the spatial intensity width, $2\sigma_s = 0.9$ mm, the transverse coherence length, $\sigma_g = 4$ mm, and the beam curvature, $R = 480$ mm. These parameters are measured for the incident beam by replacing the sample cell with a mirror and using the same heterodyne method to characterize the coherence [14,15]. For simplicity, we assume that the LO has a sufficiently small longitudinal coherence length to directly select the path length in the medium. Thus, the Dt terms in Eq. (2) are replaced by $Dt = \frac{1}{3}vl^*t = \frac{1}{3}l^*\Delta/n_0$.

Performing the tranverse position and momentum integrals in both the **x** and **y** directions, the mean square beat signal is given by

$$S_B(d_x, p_x, t) \propto \frac{\delta^2}{Dt} \exp\left(-\frac{d_x^2}{a'^2}\right) + \frac{\delta'^2}{Dt} \times \exp\left(-\frac{d_x^2}{a^2} - \delta'^2 p_x^2\right), \qquad (5)$$

where we hold the **y** components of the momentum and position equal to zero. As in Eq. (2), the first and second terms are the diffuse and weakly localized components, respectively. Here, $p_x = kd_p/f$, $1/a^2 = 1/(4\sigma_s^2) + 1/\sigma_g^2$, $1/\delta^2 = 1/(Dt) + 4/a^2$, $a'^2 = a^2(Dt/\delta^2)$, and $1/\delta'^2 = 1/\delta^2 + 4k^2\sigma_s^2/R^2$. Figures 2(e) and 2(f) show the predicted phase-space distributions for vt of 0.15 and 0.44 mm. The diffuse component produces the broad vertical bands, while the weakly localized component produces the bright central feature. In both cases, the diffusion approximation adequately reproduces the measured phase-space distribution without free parameters. For $vt = 7\ \mu$m [Fig. 2(d)], the prediction only qualitatively reproduces the measured distribution, because the diffusion model requires $vt \gg l$, which is not satisfied.

Surprisingly, neither the measured nor predicted phase-space distributions exhibit wave front curvature. When curvature is detected, the phase-space contours depend on momentum through the variable $p - kx/R$. Hence, the contours rotate due to the correlation between the momentum and position for a diverging or converging beam [12]. This rotation is readily observable when the incident beam, which has a curvature of $R = 480$ mm, is reflected from a mirror [14]. By contrast, the localized component of the backscatter is time reversed, i.e., $W(x,p) \rightarrow W(x,-p)$, and the scattered field curvature is equal and opposite to that of the incident beam, and also to that of the LO in our current experiments. For this case, the beat amplitude is readily shown to factorize into a product of Gaussian functions of p and x. Hence, there is no correlation between x and p. Thus, the lack of detected wave front curvature is a consequence of time reversal and phase conjugation in weak localization [26].

In conclusion, we have demonstrated measurement of time-resolved optical phase-space distributions in an important model problem, weak localization of light backscattered from a disordered medium. The measured phase-space contours provide a visual as well as quantitative method for describing weak localization. We have shown that backscattering is naturally described in the language of Wigner phase-space distributions, which elucidate the spatial and angular behavior of the weakly localized and unlocalized components of the backscattered light. Further experiments with a "dual LO" heterodyne method [27] will enable independent control of the position and momentum resolution and a direct observation of the reversal of the wave front curvature in weak localization. This method will also enable measurement of time-resolved Wigner phase-space distributions for individual coherence areas, leading to new insights into the propagation of coherence in random media.

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