

Loading dynamics of CO₂ laser traps

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 (Received 20 July 2000; published 6 March 2001)

We present a simple Fokker-Planck model for the evolution of the number of atoms and the spatial distribution in a CO₂ laser trap which is loaded from a magneto-optical trap (MOT). Deep CO₂ laser traps can achieve very high densities, but reach equilibrium slowly compared to shallow traps. Equilibrium times range from milliseconds for shallow traps to seconds in traps which are deep compared to the thermal energy. A universal function for the maximum number of trapped atoms is given, assuming that the optical dipole trap reaches thermal equilibrium with the MOT.

DOI: 10.1103/PhysRevA.63.043403

PACS number(s): 32.80.Pj

I. INTRODUCTION

Far-off resonance optical dipole traps (FORTs) [1] have been widely explored as a means for tightly confining atoms [2]. The simplest optical dipole trap consists of a focused red-detuned laser beam. Atoms are attracted to the maximum of the intensity, providing a confining force in three dimensions. Optical traps based on CO₂ lasers are particularly useful, as they provide a nearly conservative, state independent potential suitable for trapping both atoms [3] and molecules [4]. Since the infrared frequency is low compared to atomic resonance frequencies for transitions originating from the ground state, a focused CO₂ laser beam is a quasioleostatic trap or QUEST [3]: The depth of the attractive potential arises essentially from the ground state static polarizability. As a result of the extreme detuning from resonance and the very low infrared frequency, optical scattering in CO₂ laser traps occurs at a rate measured in photons per hour [3], so that optical heating is negligible. CO₂ lasers are also inherently stable, minimizing residual heating from laser beam intensity fluctuations and pointing noise [5,6].

CO₂ laser traps are versatile, since a variety of atomic and molecular species are readily confined for long time periods. A CO₂ laser has been used to produce a Cs optical lattice with a long spatial period [7]. An ultrastable CO₂ laser trap has been used to store fermionic ⁶Li [8]. Currently, this trap achieves a 1/e lifetime of 370 s limited by background collisions at a pressure $\approx 10^{-11}$ Torr. This trap has been used to confine stable two-state mixtures of ⁶Li to achieve rapid evaporative cooling as a result of a very large elastic cross section [9]. This system has potential applications to studies of collective oscillations and superfluidity in a two-component Fermi gas. Recently, a commercial CO₂ laser has been used to trap Cs with a trap 1/e lifetime of ≈ 150 s [10]. This has enabled studies of evaporative cooling and inelastic processes for both hyperfine states. Simultaneous trapping of Li will enable studies of interactions between multiple species, heteronuclear photoassociation, and sympathetic cooling. CO₂ laser traps also have been suggested for confining molecules excited to super-high rotational angular momentum states [11], for quantum computing based on cavity QED [12], and for precision measurements of electric dipole moments (EDM's) in atoms [13].

For many applications, it is important to maximize the number of atoms loaded into the optical trap. Previously, we determined the equilibrium number of atoms expected for a gaussian CO₂ laser trap which is continuously loaded from a magneto-optical trap (MOT), assuming that the trap comes into thermal equilibrium with the MOT [8,14]. Recently, Kuppens *et al.*, [15] have presented a thorough experimental study and a model of the dynamics for loading ⁸⁵Rb into an optical dipole trap which is detuned from resonance up to a few nanometers. In this case, the trap depth can far exceed the temperature of the MOT, and the optical trap induces large spatially dependent shifts in the atomic resonance frequency, significantly affecting the MOT operation. A general rate equation model containing a number of experimentally determined parameters is found to reproduce the data. The model includes the decay of the MOT number during the precooling phase of the loading as well as density dependent loss processes, such as radiatively induced escape when the MOT light is present, and ground state hyperfine changing collisions in the optical dipole trap [15]. However, to our knowledge, a microscopic treatment of the spatial loading dynamics, including the forces exerted by the optical dipole trap on the MOT atoms during the loading phase, has not been developed previously.

The purpose of this paper is to describe a simple model of the spatial loading dynamics of a CO₂ laser trap, as well as the equilibrium number of trapped atoms. This model takes the form of a Fokker-Planck equation which explicitly includes the forces exerted by the optical dipole trap and the viscous damping forces of the MOT. In this model, we assume that spatially dependent changes in the atomic resonance frequency arising from the presence of the CO₂ laser trap can be neglected, so that the operation of the MOT is not strongly affected. Further, we neglect attenuation of the MOT beams in the CO₂ laser trap and assume that the viscous damping coefficient and MOT temperature are spatially constant. We also neglect the effects of density dependent loss rates, such as hyperfine changing collisions and radiative escape [16]. Using the simple Fokker-Planck model, we find that the equilibrium density in the trap can be greatly enhanced compared to that of the MOT when the trap depth U_0 is large compared to the thermal energy $k_B T$. This is a consequence of the Boltzmann factor $\exp[U_0/(k_B T)]$. However, the time scale to achieve equilibrium can be quite long. De-

spite the simplicity of the model, the equilibrium number of atoms is found to agree with our measurements for trap depths ranging from less than the thermal energy to more than twice the thermal energy and the predicted time to reach equilibrium is consistent with our observations. The model provides a first step in developing a microscopic treatment of the loading dynamics and can be modified in the future to include the additional processes which have been neglected.

II. THEORY

The CO₂ laser trap is a particularly simple system for investigating loading dynamics compared to most red-detuned optical traps. In the latter case, the ground atomic state is attracted to the trap while the excited state is repelled and the corresponding light shifts induced by the trap seriously affect the operation of the MOT from which the optical trap is loaded. This somewhat complicates the loading process [15,17]. By contrast, both the excited and atomic ground states are attracted to the focus of a CO₂ laser trap, since the infrared frequency is small compared to excited state resonance frequencies. Hence, for moderate intensities, the trap-induced light shift of the optical transition frequency can be small compared to the MOT detuning, so that operation of the MOT is not significantly affected. For example, the excited and ground state static polarizabilities of ⁶Li differ by only 20%. For a well depth of 400 μK, the MOT is detuned from resonance by at most 2 MHz [8]. In our model, the MOT serves as a reservoir in thermal and diffusive contact with the CO₂ laser trap. The optical molasses of the MOT laser fields provides the viscous damping needed to cool the atoms which are pulled into the CO₂ laser trap potential.

The conservative part of the potential for the trapped atoms arises from the MOT and from the focused CO₂ laser beam:

$$U(\rho, z) = \frac{k_B T}{R^2} (\rho^2 + z^2) - U_0 \exp\left(-\frac{\rho^2}{a^2} - \frac{z^2}{z_0^2}\right). \quad (1)$$

The first term describes the effective harmonic restoring force of the MOT in the radial ρ and axial z directions. It includes both the MOT laser forces and the atom-atom radiative repulsion forces [18] that determine the MOT radius R , i.e., the effective MOT restoring force is reduced at high MOT density. For simplicity, the MOT is taken to be spherically symmetric with a radius R which can be estimated experimentally. This term yields a Boltzmann factor for the equilibrium MOT density which is a Gaussian distribution of radius R in the absence of the CO₂ laser trap. The second term describes the CO₂ laser trap, where a is the intensity $1/e$ radius and $z_0 = 2\pi a^2/\lambda$ is the Rayleigh length. We assume $a \ll R$ and neglect atom-atom radiative repulsion forces in the CO₂ laser trap. Here we approximate the Lorentzian dependence on the axial position z by a Gaussian dependence on z as used previously to simplify the form for the density of states in three dimensions [8,14]. U_0 is the well depth, which is proportional to the static polarizability and to the maximum intensity [8]. For simplicity, we also assume that the ground and excited state polarizabilities are nearly identical

so that the force on the atoms does not change when the atoms are excited in the MOT laser beams. As noted above, this is approximately the case for Li [8].

In addition to the conservative potential, the atoms experience viscous damping in the MOT optical molasses which provides cooling. The local momentum distribution is damped very rapidly according to $\dot{p} = -\alpha v$. We assume that the viscous damping coefficient α is spatially constant in the CO₂ laser trap. Then, $\dot{v} = -\gamma_v v$, where $\gamma_v = \alpha/M$, so that an atom loses memory of its velocity after traveling a distance v/γ_v . For the ideal MOT detuning and intensity in the trap region, the maximum value of α for simple Doppler cooling is $\hbar k^2/2$ [19,20], so that $\gamma_v \approx \epsilon_{\text{rec}}/(\hbar)$. For lithium, γ_v is $2\pi \times 70$ kHz. Hence, at MOT temperatures of 0.2 mK, where the velocity is of order 1 m/s, the atom travels only a few microns before losing memory of the velocity. Since this distance is small compared to diameter of the CO₂ laser trap, $2a \approx 100$ μm, we can assume the momentum distribution is in local equilibrium.

In this case, the position distribution, $n(\mathbf{x}, t)$, will obey approximately a Fokker-Planck equation [21,22],

$$\frac{\partial n(\mathbf{x}, t)}{\partial t} = \frac{1}{\alpha} \nabla \cdot [n(\mathbf{x}, t) \nabla U(\mathbf{x}) + k_B T \nabla n(\mathbf{x}, t)], \quad (2)$$

where $D_x = k_B T/\alpha$ is the spatial diffusion constant and T is the temperature. In Eq. 2, the right-hand side is $-\nabla \cdot \mathbf{J}$, where \mathbf{J} is the sum of the drift and diffusion currents. The local drift velocity is determined by the spatially varying trap force and the viscous damping coefficient. In equilibrium, the drift and diffusion currents balance and the solution to Eq. 2 is the usual Boltzmann factor. To lowest order, the phase space density $W(\mathbf{x}, \mathbf{p}, t)$ is given by the product

$$W(\mathbf{x}, \mathbf{p}, t) = n(\mathbf{x}, t) \frac{\exp[-\mathbf{p}^2/(2Mk_B T)]}{(2\pi M k_B T)^{3/2}}, \quad (3)$$

where the momentum dependent factor is a normalized Maxwellian distribution. The number of trapped atoms at time t is then obtained by integrating the phase space density over the region where the total energy is negative, i.e., between $-U_0$ and 0, for the potential of Eq. (1).

$$N_T(t) = \int d^3 \mathbf{x} d^3 \mathbf{p} \Theta\left[-\frac{\mathbf{p}^2}{2M} - U(\mathbf{x})\right] W(\mathbf{x}, \mathbf{p}, t), \quad (4)$$

where $\Theta(E)$ is a unit step function.

We consider for simplicity the case of radial loading and assume that the Rayleigh length of the trap is larger than the radius of the MOT, $z_0 \gg R$. In this case, the z dependence of the spatial distribution remains essentially unchanged from that of the MOT as discussed below. Then, the effective potential of Eq. (1) is approximated by

$$U(\rho, z) = \frac{k_B T}{R^2} (\rho^2 + z^2) - U_0 \exp\left(-\frac{\rho^2}{a^2}\right). \quad (5)$$

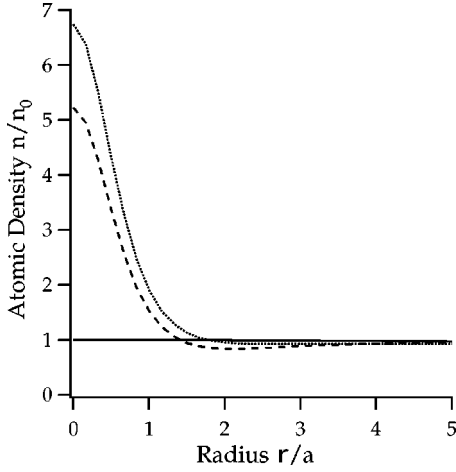


FIG. 1. Predicted radial spatial distribution $n(\rho, t)$ for $U_0 = 2k_B T$, $a = 35 \mu\text{m}$, and $\alpha = 0.1\hbar k^2$. Solid line: $t = 0$ ms. Dashed line: $t = 1$ ms. Dotted line: $t = 5$ ms.

Here, we assume the MOT provides the dominant axial confining potential for the trapped atoms while the CO₂ laser provides the dominant radial potential in the trap assuming $R \gg a$. To describe the radial motion, the Fokker-Planck equation, Eq. (2), is written in terms of the radial coordinate ρ ,

$$\frac{\partial n(\rho, t)}{\partial t} = \frac{1}{\alpha} \frac{1}{\rho} \frac{\partial}{\partial \rho} \left[\rho \left(\frac{\partial U}{\partial \rho} n + k_B T \frac{\partial n}{\partial \rho} \right) \right]. \quad (6)$$

Equation (6) is integrated numerically using Eq. (5) with $z = 0$ and $n(\rho, t=0) = n_0 \exp(-\rho^2/R^2)$ to determine the evolution of the radial density distribution of atoms in the CO₂ laser trap. Retaining the radial part of the MOT potential assures that the total number of atoms is conserved. Figure 1 shows the radial density distribution, $n(\rho, t)$ for three different times. We choose parameters comparable to those for our trap [8]: $R = 1$ mm, $a = 35 \mu\text{m}$, $U_0 = 0.33$ mK, $k_B T = 0.2$ mK, and a saturation parameter of 0.1, so that $\alpha/M \approx 0.1\hbar k^2/M = 2\pi \times 14$ kHz. The density in the center builds up as atoms are pulled in through the optical molasses by the radial force of the CO₂ laser trap.

Equation (4) can be used to determine the total number of atoms which are trapped at any time t . Since the potential given by Eq. (5) is separable, i.e., for $z_0 \gg R$, the CO₂ laser radial potential does not exert an axial force, we assume that the axial spatial distribution is in equilibrium at all times, i.e., the density scales as $\exp(-z^2/R^2)$. Then the phase space distribution takes the form

$$W(\rho, z, \mathbf{p}, t) = n(\rho, t) \frac{\exp[-z^2/R^2 - \mathbf{p}^2/(2Mk_B T)]}{(2\pi M k_B T)^{3/2}}. \quad (7)$$

Using Eq. (7), Eq. (4) can be rewritten as

$$N(t) = \int d^2 \mathbf{x}_\perp d^2 \mathbf{p}_\perp n(\rho, t) \frac{\exp[-\mathbf{p}_\perp^2/(2Mk_B T)]}{(2\pi M k_B T)^{3/2}} \times \int dE_z \Theta[-E_\perp(\rho, \mathbf{p}_\perp) - E_z] \frac{2\pi}{\omega_z} \exp\left[-\frac{E_z}{k_B T}\right], \quad (8)$$

where the MOT radial potential is negligible for the trapped atoms so that $E_\perp(\rho, \mathbf{p}_\perp) = \mathbf{p}_\perp^2/(2M) - U_0 \exp(-\rho^2/a^2)$. We have used the density of states for a one-dimensional harmonic oscillator to obtain

$$\int dz dp_z \delta\left(E_z - \frac{p_z^2}{2M} - k_B T \frac{z^2}{R^2}\right) = \frac{2\pi}{\omega_z}.$$

Here, $\omega_z = \sqrt{2k_B T/(MR^2)}$ is the oscillation frequency corresponding to the effective MOT axial potential, $k_B T z^2/R^2$.

The rest of the integrals in Eq. (8) can be straightforwardly done to obtain

$$N(t) = N_0 \int_0^\infty \frac{2\rho d\rho}{a^2 n_0} n(\rho, t) f[\rho, U_0/(k_B T)], \quad (9)$$

where

$$f(\rho, q) = 1 - (1 + q \exp(-\rho^2/a^2)) \times \exp(-q \exp(-\rho^2/a^2)). \quad (10)$$

The number of trapped atoms is given in terms of the number N_0 contained in the overlap volume of the trap with the MOT,

$$N_0 = \pi^{3/2} a^2 R n_0, \quad (11)$$

where n_0 is the MOT central density.

Using Eq. (9), the number of atoms $N(0)$ initially contained in the trap is obtained by setting $n(\rho, t=0) = n_0$, the initial MOT density at $\rho = z = 0$. One obtains

$$N(0) = N_0 q^2 \int_0^1 dv v (-\ln v) \exp(-qv), \quad (12)$$

where $q = U_0/(k_B T)$. Note that the initial number is equivalent to the number of atoms trapped by turning the CO₂ laser trap on suddenly, since the momentum distribution is assumed to be instantly in equilibrium. Figure 2 shows that the maximum initial number is the order of N_0 and increases slowly for large $U_0/(k_B T)$ due to increase in the volume for which the negative potential is comparable to $k_B T$.

The equilibrium number of trapped atoms $N(t \rightarrow \infty)$ is obtained using $n(\rho, \infty) \approx n_0 \exp[+q \exp(-\rho^2/a^2)]$, assuming $R \gg a$. Then,

$$N(\infty) = N_0 q \int_0^1 dv (-\ln v) [\exp(qv) - 1]. \quad (13)$$

Note that for very shallow traps, $U_0/(k_B T) \ll 1$, $N(\infty) = N(0)$. Figure 2 compares the equilibrium number of

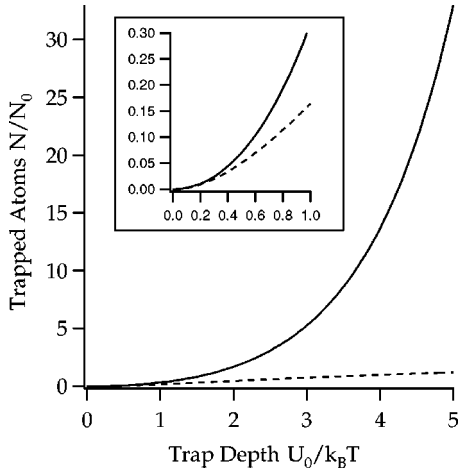


FIG. 2. Predicted number of trapped atoms N at $t=0$ (dashed line) and in equilibrium (solid line) as a function of $U_0/(k_B T)$, where U_0 is the well depth and $k_B T$ is the thermal energy. N is given in units of the number N_0 contained in the trap volume at the MOT density.

trapped atoms to the initial number as a function of $q = U_0/(k_B T)$. For deep traps, where $q = U_0/(k_B T) \gg 1$, the exponential in the integrand of Eq. (13) leads to substantial density enhancement. This may account in part for the density enhancement observed in Refs. [15,17].

The time scale for the trap to reach equilibrium increases with $U_0/(k_B T)$ as shown in Fig. 3. For large values of $U_0/(k_B T)$, the equilibrium trap number is large compared to N_0 , the number contained in the trap volume at the initial MOT density. Atoms must drift in from large ρ to fill the trap fully. Since the force is weak when $\rho \gg a$, and the optical molasses of the MOT provides a high viscosity medium, this time scale can be several seconds when $U_0/(k_B T)$ is large. This may account in part for the long time scales observed in Ref. [15]. Note that for fixed $q = U_0/(k_B T)$ and fixed R/a , the equilibrium time scales with the diffusion time, $\tau \approx \alpha a^2/(k_B T)$.

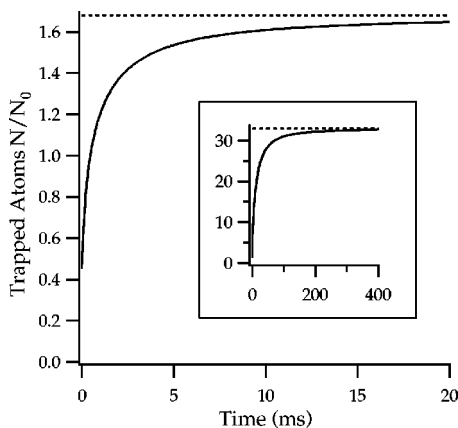


FIG. 3. Predicted number of trapped atoms $N(t)$ as a function of time for a well depth $U_0 = 2k_B T$. Inset (0–400 ms) shows $N(t)$ for a deep trap with $U_0 = 5.0k_B T$. $N(t)$ is given in units of the number N_0 contained in the trap volume at the MOT density.

TABLE I. Number of trapped atoms.

U_0 (mK)	N_{meas}	N_{calc}	
		$T=0.2$ mK	$T=0.15$ mK
0.33	4.0×10^5	3.8×10^5	8.1×10^5
0.1	4.4×10^4	2.6×10^4	4.8×10^4

III. EXPERIMENT

It is interesting to compare the equilibrium number of trapped atoms predicted by Eq. (13) to that obtained in our CO_2 laser trap. In the experiments, the MOT is loaded for 3 s and then cooled to near the Doppler limit by reducing the MOT laser intensity and tuning the laser closer to resonance for up to 30 ms. Using time-of-flight measurements, the temperature is estimated to be between 0.15 and 0.2 mK. The MOT radius is measured to be $R \approx 1.0$ mm in the axial direction both before and after the cooling phase. The MOT density in both phases is estimated to be $n_0 \approx 0.5 \times 10^{11}$ atoms/cm³ by absorption imaging. The trap radii are $a = 35$ μm and $b = 47$ μm . The Rayleigh length is $z_0 = 0.8$ mm, smaller than R . However, the trap loading time in the final cooling stage is limited to a maximum of 30 ms, short compared to the axial equilibration time in the trap. Hence, the z dependence of the density cannot change quickly and the faster radial motion dominates the density enhancement. Thus, the radial loading model is approximately valid. The primary error is that the radial force is not constant over the Rayleigh length. We take $N_0 = \pi^{3/2} ab z_0 n_0 = 3.7 \times 10^5$ as the number of atoms in the overlap volume. Table I shows the measured number of trapped atoms and the predictions of Eq. (13) for two well depths, 0.1 and 0.33 mK. Equation (13) is evaluated for temperatures of 0.15 and 0.2 mK, and is in reasonable agreement the measurements. At a well depth of 0.33 mK, the measured loading time is optimized at 4 ms for a saturation parameter of 0.1 with the detuning optimized to give a minimum temperature near the Doppler limit. Increasing the loading time to 30 ms has no effect, while lowering it to 1 ms reduces the atom number. Figure 3 shows that for $U_0/(k_B T) \approx 2$ and $\alpha = 0.1 \hbar k^2$, the expected equilibrium time is ≈ 4 ms, in very good agreement with the measurements.

IV. CONCLUSIONS

In conclusion, we have presented a simple Fokker-Planck equation treatment of the loading dynamics of a CO_2 laser trap. We predict that deep traps can achieve very high densities, but can take much longer to fill than shallow traps. The predictions for the number of trapped atoms at modest trap depths are in good agreement with our measurements and the equilibrium time scales are consistent with our observations.

Our model is not completely general, as we have neglected a variety of processes that can affect the loading dynamics. For example, at sufficiently high trap intensity, the difference in the excited and ground state polarizabilities will

lead to significant position-dependent shifts in the atomic resonance frequency, affecting the operation of the MOT in the trap region.

We have also neglected absorption of the MOT beams in the high density region of the CO₂ laser trap. Absorption causes the viscous damping coefficient α and the momentum diffusion coefficient D_p to be position dependent. A mitigating factor is that the temperature may be spatially constant over a substantial radius, since $k_B T = D_p / \alpha$, and both D_p and α are proportional to the intensity of the MOT beams at low intensity. Attenuation of the MOT beams also may suppress density-dependent radiatively induced inelastic loss and radiative atom-atom repulsion in the CO₂ laser trap, enabling increased density. In this case, the trap may function

analogously to a dark MOT [23], except that the confining force is provided by the laser trap.

The model can be readily improved by including the position dependence of the viscous damping coefficient. In addition, a two-state Fokker-Planck equation can be used to describe inelastic collisions between the upper and lower hyperfine levels which expel atoms from the trap. When the MOT beams are attenuated, hyperfine state changing collisions may dominate the optical pumping rates, and favor loading into the stable lower hyperfine level at high density. The current treatment will serve as a starting point for including these and other features in future work.

This work has been supported by the U. S. Army Research Office and the National Science Foundation.

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